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Original Paper

The Parametric and Non-parametric Bootstrap Resamplings for the Visual Acuity Measurement

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Abstract

We propose a useful tool for the visual acuity measurement from the results of parametric and non-parametric bootstrap algorithms in the logistic regression model. We present the kurtosis and the variance of deviance residuals to estimate the efficiency of bootstrap resampling. We applied our parametric and non-parametric algorithms to the problem of the visual acuity measurement and obtained the efficiency measures for the comparison of the parametric and non-parametric bootstrap resamplings.

1. Introduction

Mita et al. [1] developed a statistical method for evaluating logarithmic visual acuity by logistic regression. In the paper [2] Mita et al. proposed the non-parametric bootstrap resampling algorithm for visual acuity measurement. Bach [3] and Schulze-Bonsel et al. [4] developed the automated procedures on the personal computer for measurements of visual acuity. Nagai et al. [5] proposed the statistical significance testing of difference between multiple thresholds by the constant stimuli method.

The bootstrap resampling technique provides a powerful procedure for estimating the variance of a parameter of a function. For the problems of the regression model we can refer to Habing [6], Hossain et al. [7] and Rashid [8]. Amiri et al. [9] showed that the performance of the parametric and non-parametric bootstrap for variance estimation depended on the sample kurtosis and on the kurtosis of distribution used to generate the bootstrap observations in the parametric method.

In the present paper we propose the parametric bootstrap resampling and show the comparison of parametric and non-parametric bootstrap results. We summarize our algorithms of [1] and [2] in which we adopt the logistic regression with the guessing rate in section 2 and the log-likelihood ratio test statistics of two-sample problems in section 3. We introduce the parametric and non-parametric bootstrap resamplings in section 4. Then we present the kurtosis and the variance of deviance residuals for the parametric and non-parametric resamplings in section 5. Finally, in section 6 we show an application of our parametric and non-parametric bootstrap algorithms to the visual acuity measurement.

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2. Logistic regression and deviance residuals

We assume that the probability p is defined by

$$p\left(x;\alpha,\beta,\gamma_0\right)=p_0\left(x;\alpha,\beta\right)+\gamma_0\left(1-p_0\left(x;\alpha,\beta\right)\right),$$

where

$$p_0\left(x;\alpha,\beta\right)=\left(1+\exp\left(-\alpha-\beta x\right)\right)^{-1},$$

x is the explanatory variable, α and β are unknown parameters, γ_0 $(0 \leq \gamma_0 < 1)$ is a known constant which defines the guessing rate.

Let X be the set of binomial observations such that

$$X=\{x_j,\mu_j \ \ (j=1,2,\cdots,N)\},$$

where x_j and μ_j $(j=1, 2, \cdots, N)$ are the explanatory variables and the outcome data respectively for j-th $(j=1, 2, \cdots, N)$ observations. μ_j $(j=1, 2, \cdots, N)$ are defined by

$$\mu_j=\begin{cases}1 & \text{if} \quad j\text{-th outcome is “success“}, \\ 0 & \text{if} \quad j\text{-th outcome is “failure“}.\end{cases}$$

The binomial likelihood $\ell(\alpha, \beta, \gamma_0)$ is given by

$$\ell(\alpha,\beta,\gamma_0)=\prod_{j=1}^{N} p_j^{\mu_j}(1-p_j)^{1-\mu_j},$$

where $p_j = p(x_j ; \alpha, \beta, \gamma_0)$ $(j=1, 2, \cdots, N)$. Then we can obtain the optimal values $\hat{\alpha}$ and $\hat{\beta}$ for α and β respectively by adopting the Fisher score method. By using $\hat{\alpha}$ and $\hat{\beta}$ we can compute the deviance D and the deviance residuals ε_j^0 $(j=1, 2, \cdots, N)$ such that

$$D=-2\log\hat{\ell},$$

$$\varepsilon_j^0=\begin{cases}-\sqrt{2\log\dfrac{1}{1-\hat{p}_j}} & \text{if} \quad \mu_j=0, \\ \sqrt{2\log\dfrac{1}{\hat{p}_j}} & \text{if} \quad \mu_j=1\end{cases}$$

$$(j=1,2,\cdots,N),$$

where

$$\hat{\ell}=\prod_{j=1}^{N}\hat{p}_j^{\mu_j}(1-\hat{p}_j)^{1-\mu_j},$$

$$\hat{p}_j=\hat{p}_0(x_j)+\gamma_0(1-\hat{p}_0(x_j)),$$

$$\hat{p}_0(x_j)=(1+\exp(-\hat{\alpha}-\hat{\beta}x_j))^{-1}$$

$$(j=1,2,\cdots,N).$$

3. Log-likelihood ratio test statistics of two-sample problems

Let X_1 and X_2 be the samples taken from the populations which have possibly different probability distributions Φ_1 and Φ_2 respectively. We shall test the following hypothesis:

$$\text{null hypothesis } \ H_0 : \Phi_1=\Phi_2,$$

$$\text{alternative hypothesis } \ H_1 : \Phi_1\neq\Phi_2.$$

Let X_3 be the combined sample of X_1 and X_2 :

$$X_3=X_1\bigcup X_2.$$

Then we have the log-likelihood ratio test statistics G for the two-sample test:

$$G = D_3 - (D_1 + D_2),$$

where D_k ($k = 1, 2, 3$) are the deviances for samples X_k ($k = 1, 2, 3$) respectively.

4. Parametric and non-parametric bootstrap resamplings of deviance residuals

Let ε^0 be the set of deviance residuals ε_j^0 ($j = 1, 2, \cdots, N$) obtained by logistic regression such that

$$\varepsilon^0 = \left\{\varepsilon_j^0 \ \ (j = 1, 2, \cdots, N)\right\}.$$

Let $\bar{\varepsilon^0}$ and $\mathrm{sd}(\varepsilon^0)$ be the mean and standard deviation of ε^0.

(i) Parametric bootstrap resampling

For the sufficiently large integer B (= number of bootstraps), let ζ_j^b ($j = 1, 2, \cdots, N$; $b = 1, 2, \cdots, B$) be the random numbers which are generated by the Box-Muller method (refer to [10]). Then the distribution of ζ_j^b ($j = 1, 2, \cdots, N$) for a specific b ($1 \leq b \leq B$) is approximately equal to $N(0, 1)$, where $N(0, 1)$ is the normal distribution with mean equalling to 0 and standard deviation equalling to 1. We define the parametric bootstrap resampling $\varepsilon^{\#}$ of deviance residuals such that

$$\varepsilon^{\#} = \bigcup_{b=1}^{B} \varepsilon^{\#b},$$

$$\varepsilon^{\#b} = \left\{\varepsilon_j^{\#b} \ \ (j = 1, 2, \cdots, N)\right\} \qquad (b = 1, 2, \cdots, B),$$

where

$$\varepsilon_j^{\#b} = \bar{\varepsilon^0} + \zeta_j^b \cdot \mathrm{sd}(\varepsilon^0) \qquad (j = 1, 2, \cdots, N; b = 1, 2, \cdots, B).$$

(ii) Non-parametric bootstrap resampling

Let I_k ($k = 1, 2, \cdots, N$) be the intervals defined by

$$I_k = \left(\frac{k-1}{N}, \frac{k}{N}\right) \quad (k = 1, 2, \cdots, N).$$

We set the one-to-one relationship between the deviance residuals ε_k^0 and the intervals I_k for common index $k = 1, 2, \cdots, N$.

For the sufficiently large integer B (= number of bootstraps), let u_j^b ($j = 1, 2, \cdots, N$; $b = 1, 2, \cdots, B$) be the uniform random numbers chosen from the interval (0 , 1).

Then we define the non-parametric bootstrap resampling ε^* of deviance residuals such that

$$\varepsilon^* = \bigcup_{b=1}^{B} \varepsilon^{*b},$$

$$\varepsilon^{*b} = \left\{\varepsilon_j^{*b} \equiv \varepsilon_k^0 \ \ \text{s.t. } u_j^b \in I_k \ \ (j = 1, 2, \cdots, N; 1 \leq k \leq N)\right\} \qquad (b = 1, 2, \cdots, B).$$

5. Kurtosis and variances of deviance residuals

Let $K(Z)$ be the kurtosis of a data set $Z = \{z_j\ (j = 1, 2, \cdots, N)\}$ defined by

$$K(Z) = \frac{k_4}{(k_2)^2} - 3,$$

where

$$k_4 = \frac{1}{N}\sum_{j=1}^{N}(z_j - \bar{z})^4,$$

$$k_2 = \frac{1}{N}\sum_{j=1}^{N}(z_j - \bar{z})^2,$$

$$\bar{z} = \frac{1}{N}\sum_{j=1}^{N} z_j.$$

Let $K^0 = K(\varepsilon^0)$ be the kurtosis of the deviance residuals $\varepsilon^0 = \{\varepsilon_j^0 (j = 1, 2, \cdots, N)\}$ of logistic regression.

Let $K^\#$ and K^* be the kurtosis of parametric and non-parametric bootstrap resamplings $\varepsilon^\#$ and ε^* respectively defined by

$$K^\# = \frac{1}{B}\sum_{b=1}^{B} K(\varepsilon^{\# b}),$$

$$K^* = \frac{1}{B}\sum_{b=1}^{B} K(\varepsilon^{* b}),$$

where

$$\varepsilon^{\# b} = \left\{\varepsilon_j^{\# b} \ \ (j = 1, 2, \cdots, N)\right\},$$
$$\varepsilon^{* b} = \left\{\varepsilon_j^{* b} \ \ (j = 1, 2, \cdots, N)\right\}$$
$$(b = 1, 2, \cdots, B).$$

Let $V(Z)$ be the variance of a data set $Z = \{z_j \ (j = 1, 2, \cdots, N)\}$ in the sense of the maximum likelihood estimator such that

$$V(Z) = \frac{1}{N}\sum_{j=1}^{N}(z_j - \bar{z})^2,$$

where

$$\bar{z} = \frac{1}{N}\sum_{j=1}^{N} z_j.$$

Let $V^\#$ and V^* be the variances of parametric and non-parametric bootstrap resamplings $\varepsilon^\#$ and ε^* respectively defined by

$$V^\# = \frac{1}{B}\sum_{b=1}^{B} V(\varepsilon^{\# b}),$$

$$V^* = \frac{1}{B}\sum_{b=1}^{B} V(\varepsilon^{* b}).$$

We define the efficiency measure e for the comparison of parametric and non-parametric bootstrap resamplings such that

$$e = \frac{V^*}{V^\#}.$$

6. Application to visual acuity measurement

(i) Logistic regression of samples 1, 2 and 3

We use the same examples described in [2] which are summarized in the following:

We took the data (sample size N_1 = 120) from one individual with no visual abnormalities in order to assess our bootstrap algorithm. The LogVA (Logarithmic Visual Acuity) is 0.3681±0.0209 in complete refractive correction and we adopt this data set as sample 1. The data of sample 2 (N_2 = 80) is taken in +0.50D incomplete refractive correction from the same individual of sample 1. Sample 3 (N_3 = 200) is constructed by the combined data of samples 1 and 2.

Since we adopt the Landolt-C of four different orientations in our visual acuity measurement, the guessing rate γ_0 is chosen as $\gamma_0 = 0.25$. The explanatory variable x is the logarithmic visual acuity.

The logistic regression results of samples 1, 2 and 3 are shown in Table 1. Figures 1, 2, and 3 show the observed data (dots •) and $\hat{p}(x) = p(x\,;\hat{\alpha}, \hat{\beta}, \gamma_0)$ of samples 1, 2 and 3 respectively.

Table 1 Logistic regression results of samples 1, 2 and 3

	sample 1 $(k=1)$	sample 2 $(k=2)$	sample 3 $(k=3)$
N_k	120	80	200
$\hat{\alpha}_k$	6.2105	4.5209	4.4349
$\hat{\beta}_k$	−16.8720	−18.3394	−14.1192
γ_0	0.25	0.25	0.25
$\mathrm{se}(\hat{\alpha}_k)$	1.4383	1.5872	0.9012
$\mathrm{se}(\hat{\beta}_k)$	4.2411	6.6365	3.0423
$\mathrm{se}(\gamma_0)$	0.0	0.0	0.0
$\hat{\xi}_k$	0.3681	0.2465	0.3141
$\mathrm{se}(\hat{\xi}_k)$	0.0209	0.0214	0.0170
D_k^0	115.546	90.241	224.026

$G^0 = D_3^0 - (D_1^0 + D_2^0) = 18.239$

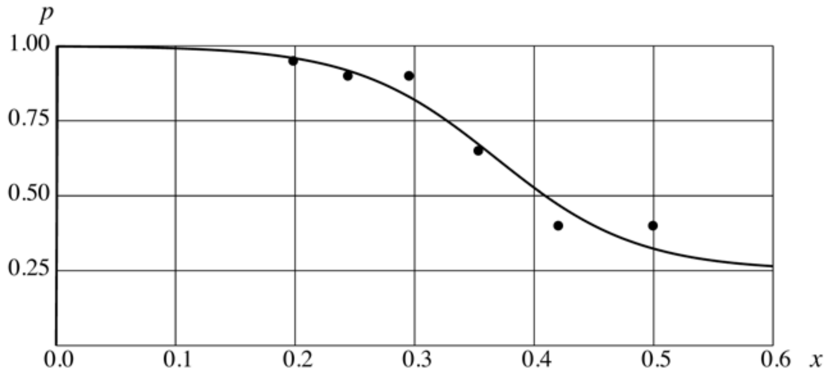

Fig. 1 Observed data (dots •) and $\hat{p}(x) = p(x\,;\hat{\alpha}, \hat{\beta}, \gamma_0)$ of sample 1

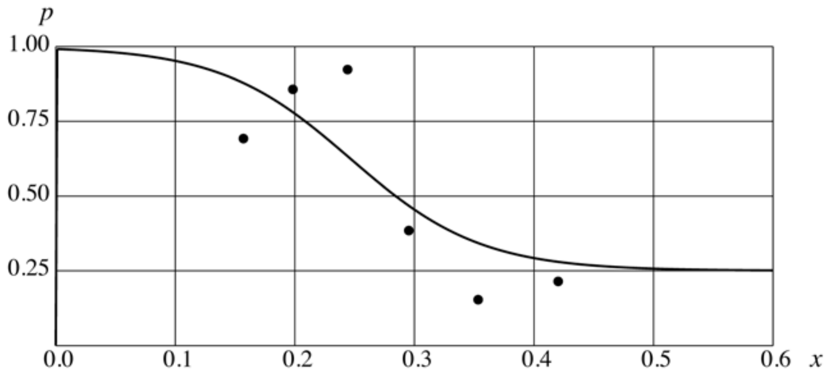

Fig. 2 Observed data (dots •) and $\hat{p}(x) = p(x\,;\hat{\alpha}, \hat{\beta}, \gamma_0)$ of sample 2

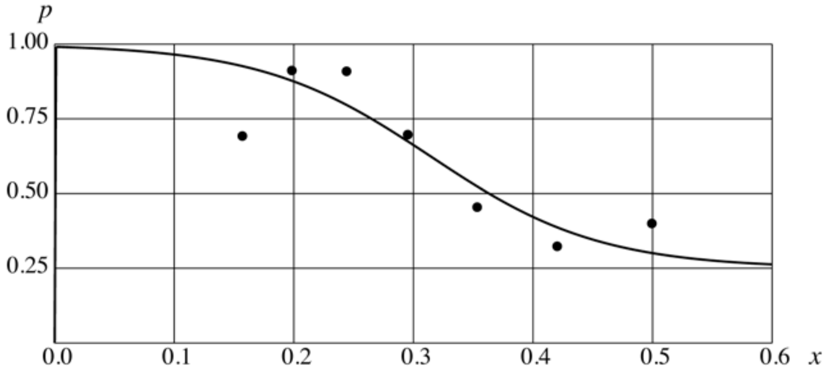

Fig. 3 Observed data (dots •) and $\hat{p}(x) = p(x ; \hat{\alpha}, \hat{\beta}, \gamma_0)$ of sample 3

(ii) The visual acuity test of the two-sample problem

We shall prove that the samples 1 and 2 are taken from the populations which have different distributions Φ_1 and Φ_2 respectively. We shall adopt the following hypothesis:

$$\text{null hypothesis } H_0 : \Phi_1 = \Phi_2,$$
$$\text{alternative hypothesis } H_1 : \Phi_1 \neq \Phi_2.$$

Let D_k^0 $(k = 1, 2, 3)$ be the deviances for samples X_k $(k = 1, 2, 3)$ respectively.

Let D_k^{*b} and $D_k^{\#b}$ $(k = 1, 2, 3; b = 1, 2, \cdots, B)$ be the deviances of non-parametric and parametric bootstraps respectively which are computed by adopting the deviance residuals ε_k^{*b} and $\varepsilon_k^{\#b}$ $(k = 1, 2, 3; b = 1, 2, \cdots, B)$ respectively.

Then we have

$$G^0 = D_3^0 - (D_1^0 + D_2^0),$$
$$G^{*b} = D_3^{*b} - (D_1^{*b} + D_2^{*b}),$$
$$G^{\#b} = D_3^{\#b} - (D_1^{\#b} + D_2^{\#b})$$
$$(b = 1, 2, \cdots, B).$$

Let ASL^* and $ASL^\#$ be the achieved significant level of non-parametric and parametric bootstraps respectively defined by

$$ASL^* = \frac{1}{B+1}(\sum_{b=1}^{B} \lambda^{*b} + 1),$$

$$ASL^\# = \frac{1}{B+1}(\sum_{b=1}^{B} \lambda^{\#b} + 1),$$

where

$$\lambda^{*b} = \begin{cases} 1 & \text{if} \quad G^{*b} \geq G^0, \\ 0 & \text{if} \quad G^{*b} < G^0, \end{cases}$$

$$\lambda^{\#b} = \begin{cases} 1 & \text{if} \quad G^{\#b} \geq G^0, \\ 0 & \text{if} \quad G^{\#b} < G^0. \end{cases}$$

Table 2 shows the results of the two-sample test by non-parametric and parametric bootstraps.

(iii) The visual acuity test of one-sample problem

In the cases of one-sample tests of α and β, we adopt the following hypothesis:

$$\text{null hypothesis } H_0 : \quad \alpha = 0,$$
$$\text{alternative hypothesis } H_1 : \quad \alpha \neq 0.$$

for α, and

$$\text{null hypothesis } \ H_0: \ \ \beta = 0,$$
$$\text{alternative hypothesis } \ H_1: \ \ \beta \neq 0.$$

for β.

In the case of threshold ξ, we adopt the following hypothesis:

$$\text{null hypothesis } \ H_0: \xi_2 = \xi_1,$$
$$\text{alternative hypothesis } \ H_1: \xi_2 \neq \xi_1.$$

for ξ_1 (sample 1) and ξ_2 (sample 2).

Let ASL^* and $ASL^{\#}$ be the achieved significant level of non-parametric and parametric bootstraps respectively.

One-sample tests by bootstrap resampling for α, β, ξ in samples 1, 2 are shown in Table 3.

Table 2 The two-sample test by bootstrap resampling

	sample 1 $(k=1)$	sample 2 $(k=2)$	sample 3 $(k=3)$
N_k	120	80	200
B	2000	2000	2000
$D_k^* = \frac{1}{B}\sum_{b=1}^{B} D_k^{*b}$	8.962	6.612	17.862
$D_k^{\#} = \frac{1}{B}\sum_{b=1}^{B} D_k^{\#b}$	8.582	6.402	17.154

$*$ shows non-parametric bootstrap

$\#$ shows parametric bootstrap

$ASL^* = \frac{1}{B+1}(\sum_{b=1}^{B} \lambda^{*b} + 1) = 0.0005$

$ASL^{\#} = \frac{1}{B+1}(\sum_{b=1}^{B} \lambda^{\#b} + 1) = 0.0005$

Table 3 One-sample test by bootstrap resampling

	sample 1 $(k=1)$	sample 2 $(k=2)$
N_k	120	80
B	2000	2000
$\hat{\alpha}_k$	6.2105	4.5209
ASL_k^*	0.0005	0.0050
$ASL_k^{\#}$	0.0005	0.0025
$\hat{\beta}_k$	-16.8720	-18.3394
ASL_k^*	0.0005	0.0095
$ASL_k^{\#}$	0.0005	0.0055
$\hat{\xi}_k$	0.3681	0.2465
ASL_k^*	-	0.0005
$\min\,(I_{0.95}^*)_k$	0.355	0.231
$\max\,(I_{0.95}^*)_k$	0.380	0.262
$ASL_k^{\#}$	-	0.0005
$\min\,(I_{0.95}^{\#})_k$	0.355	0.231
$\max\,(I_{0.95}^{\#})_k$	0.381	0.261

$*$ shows non-parametric bootstrap

$\#$ shows parametric bootstrap

(iv) Confidence interval of threshold

For the optimal parameters $\hat{\alpha}$ and $\hat{\beta}$, we define the threshold $\hat{\xi}$ with guessing rate γ_0

$$\hat{\xi} = \hat{p}^{-1}\left(\frac{1+\gamma_0}{2}\right),$$

where

$$\hat{p}(x) = \hat{p}_0(x) + \gamma_0(1-\hat{p}_0(x)),$$
$$\hat{p}_0(x) = (1+\exp(-\hat{\alpha}-\hat{\beta}x))^{-1}.$$

Then we can obtain the confidence interval I_ρ of threshold ξ in confidence coefficient ρ $(0 < \rho < 1)$ such that

$$I_\rho : \quad \hat{\xi} - \psi^{-1}\left(\frac{1-\rho}{2}\right)\cdot \mathrm{se}(\xi) \le \xi \le \hat{\xi} + \psi^{-1}\left(\frac{1+\rho}{2}\right)\cdot \mathrm{se}(\xi),$$

where $\psi(z)$ $(-\infty < z < +\infty)$ is the cumulative distribution function of threshold which is obtained by bootstrap resampling.

Table 3 shows the confidence intervals $I^*_{0.95}$ and $I^\#_{0.95}$ of threshold for the non-parametric and parametric bootstrap respectively.

(v) Kurtosis and variances of deviance residuals

Figures 4 (sample 1) and 5 (sample 2) show the distribution functions $f(z)$ of deviance residual $z = \varepsilon^0 - \bar{\varepsilon}^0$.

Table 4 shows the kurtosis and variances of deviance residuals for sample 1 ($k = 1$) and sample 2 ($k = 2$) :

$$\begin{aligned}
&K_k^0 = K(\varepsilon_k^0) : \text{kurtosis of } \varepsilon_k^0 \text{ of logistic regression,}\\
&K_k^* : \text{kurtosis of non-parametric } \varepsilon_k^*,\\
&V_k^* : \text{variance of non-parametric } \varepsilon_k^*,\\
&K_k^\# : \text{kurtosis of parametric } \varepsilon_k^\#,\\
&V_k^\# : \text{variance of parametric } \varepsilon_k^\#\\
&\qquad\qquad\qquad\qquad (k = 1, 2).
\end{aligned}$$

We can see that the non-parametric K_k^* ($k = 1, 2$) are nearly equal to K_k^0 ($k = 1, 2$) respectively. The parametric $K_k^\#$ ($k = 1, 2$) are nearly equal to 0 (which means that $K_k^\#$ ($k = 1, 2$) are nearly equal to normal distributions). The efficiency measures $e_k = V_k^*/V_k^\#$ ($k = 1, 2$) are

$$e_1 = 0.98570, \quad e_2 = 0.98005.$$

This means that the efficiencies of the non-parametric bootstraps are better than the parametric bootstraps (refer to [9]).

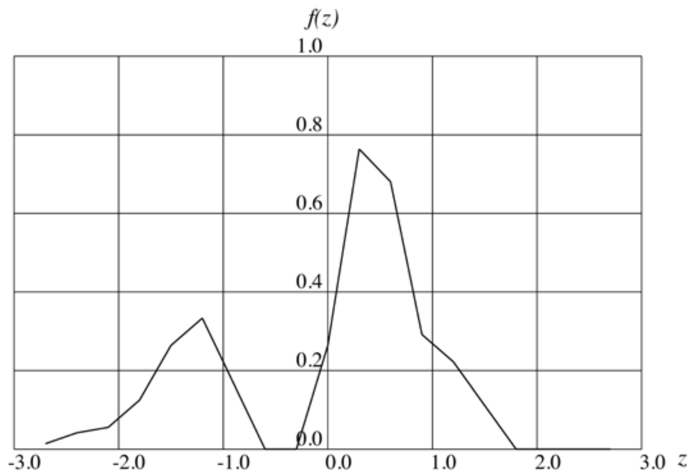

Fig. 4 The distribution function $f(z)$ of deviance residual $z = \varepsilon^0 - \bar{\varepsilon}^0$ of sample 1

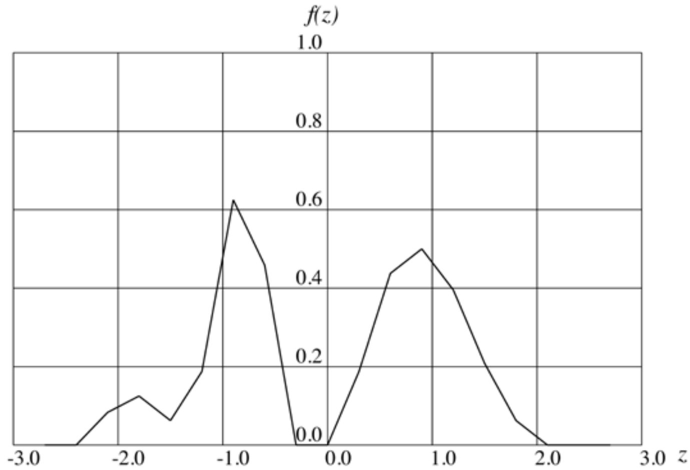

Fig. 5 The distribution function $f(z)$ of deviance residual $z = \varepsilon^0 - \bar{\varepsilon}^0$ of sample 2

Table 4 Kurtosis and variances of deviance residuals

	sample 1 $(k = 1)$	sample 2 $(k = 2)$
N_k	120	80
$K_k^0 = K(\varepsilon_k^0)$	-0.50149	-1.33653
B	2000	2000
$K_k^* = \frac{1}{B}\sum_{b=1}^{B} K(\varepsilon_k^{*b})$	-0.47966	-1.31709
$V_k^* = \frac{1}{B}\sum_{b=1}^{B} V(\varepsilon_k^{*b})$	0.94790	1.11347
$K_k^\# = \frac{1}{B}\sum_{b=1}^{B} K(\varepsilon_k^{\#b})$	-0.04495	-0.07383
$V_k^\# = \frac{1}{B}\sum_{b=1}^{B} V(\varepsilon_k^{\#b})$	0.96165	1.13614
$e = \frac{V_k^*}{V_k^\#}$	0.98570	0.98005

$*$ shows non-parametric bootstrap

$\#$ shows parametric bootstrap

7. Concluding remarks

We proposed the parametric and non-parametric bootstrap algorithms in the logistic regression model. The main properties of our algorithms are summarized as follows:

(i) logistic regression including the guessing rate,

(ii) log-likelihood test statistics of the two-sample problem,

(iii) parametric and non-parametric bootstrap resamplings of deviance residuals,

(iv) kurtosis and the variance of deviance residuals, and the efficiency measure for the comparison of parametric and non-parametric bootstrap resamplings.

We applied our algorithms to the problem of the visual acuity measurement. We obtained the following results:

(i) We find that there are few differences between the parametric and non-parametric bootstrap results in the cases of two-sample and one-sample tests.

(ii) The efficiency measures show that the non-parametric bootstraps are better than parametric bootstraps. However, the differences of efficiencies are not large.

We conclude that both parametric and non-parametric bootstrap algorithms provide useful tools for the visual acuity measurement. We shall continue to take more data from a wide range of patients and shall establish the reliable and handy tool which can be adopted by ophthalmologists everywhere.

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