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原著

Maximum Principles for Finite Element Solutions on a Riemann Surface

Hisao MIZUMOTO[1] Heihachiro HARA[2]

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Department of Medical Informatics Faculty of Medical Professions Kurashiki, 701-01, Japan[1]

Department of Information Science Faculty of Science Matsue, Shimane 690, Japan[2]

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Abstract

¥documentclass[a4paper,10pt]{jarticle} ¥pagestyle{plain} %%%%% TEXT START %%%%% ¥begin{document} In the present paper we establish the maximum and minimum principles for the finite element approximate solutions of the partial differential equation: $¥Delta u-qu=f$ on a compact bordered Riemann surface $¥overline{¥Omega}$. First we construct a triangulation K of $¥overline{¥Omega}$ with width h and introduce a class $S=S(K)$ of element functions on K. For a partition to two parts C_{1} and C_{2} of the boundary $¥partial¥Omega$, we define the finite element approximation $¥omega_{h} ¥in S$ of the boundary value problem: $¥Delta u- qu=f$ on $¥Omega$, $u=¥chi$ on C_{1} and $¥partial u/¥partial n=0$ on C_{2}, where by $¥partial u/¥partial n$ we denote the inner normal derivative of u on C_{2}. The main result in the present paper is stated as follows: For sufficiently small $h>0$, the inequality ¥[|¥omega_{h}| ¥le ¥max_{C_{1}}¥chi + ¥frac{1}{¥sin¥theta} ¥int¥int_{¥Omega}|f|dxdy ¥] holds, where $¥theta$ is the smallest value of all angles of the 2-simplices of K. The last inequality will be very useful to obtain error estinates of the finite element solutions for the theoretic ones. ¥end{document}

要 約

¥documentclass[a4paper,10pt]{jarticle} ¥pagestyle{plain} %%%%% TEXT START %%%%% ¥begin{document} この論文では,縁をもつコンパクトなリーマン面 $¥overline{¥Omega}$ 上で定義された偏微分方程式: $¥Delta u-qu=f$ の有限要素解に対する最大最小値の原理を確立する.まず,$¥overline{¥Omega}$ の幅 h の三角形分割 K を作成し,K 上の要素関数のクラス $S=S(K)$ を導入する.境界 $¥partial¥Omega$ の二つの部分 C_{1},C_{2} への分割に対して,境界値問題: $¥Omega$ 上で $¥Delta u-qu=f$, C_{1} 上で $u=¥chi$, C_{2} 上で $¥partial u/ ¥partial n=0$ の有限要素近似 $¥Omega_{h} ¥in S$ を定義する,ここで, $¥partial u/ ¥partial n$ は, u の

C_{2} 上での内法線方向微分を表す.この論文の主要結果は,つぎのように述べられる: 十分に小さい $h>O$ に対して,不等式 ¥[|¥omega_{h}| ¥le ¥max_{C_{1}}¥chi + ¥frac{1}{¥sin¥theta} ¥int¥int_{¥Omega}|f|dxdy ¥] が成り立つ,ここで $¥theta$ は K のすべての2-単体の内角の最小値である.この不等式は,有限要素解の理論解に対する誤差評価をするときに,非常に有用となるものである. ¥end{document}